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On the Effective Energy Consumption in Wireless Sensor Networks

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***Abstract*—We analytically characterize the energy consumption per successfully transmitted packet in end-to-end packet transmissions in a wireless sensor network where nodes are identically and independently distributed in a square area following a homogeneous Poisson process. It is assumed that a greedy forwarding protocol is used for routing. We obtain analytical results on the average number of hops between any two nodes if the transmission is successful and the average number of hops traversed by packets before being dropped if the transmission is unsuccessful. A transmission is *unsuccessful* if the packet sent from a source to a destination has to be dropped at an intermediate node because the greedy forwarding routing protocol is unable to find a next-hop node that is closer to the destination. Based on the above analysis, we derive the effective energy consumption per successfully transmitted packet. We show that there exists an optimum transmission range which minimizes the effective energy consumption and such optimum transmission range can be computed based on our analysis.**

I. Introduction

Minimizing energy consumption is one of the major considerations in the design of battery powered Wireless Sensor Networks (WSNs). In many scenarios it is difficult to change or recharge a battery for the sensors. The problem of minimizing energy consumption in WSNs has attracted many researchers, and various approaches for reducing energy consumption have been proposed. For example, there are a number of energy-efficient scheduling mechanisms [1] which consider scheduling strategies that switch off some sensors to save power, while maintaining a certain level of connectivity. Some researchers also considered energy-efficient routing algorithms [2].

Transmission power is an important design consideration. It affects not only the energy consumption but also other aspects of network performance. Reducing the transmission range will reduce the power consumed in transmitting a packet toward the neighbors. But reducing the transmission range will also reduce the probability of successful transmissions and increase the number of hops between a given source-destination pair. Previous studies on energy minimizing transmission power mainly consider the minimum transmission power required to maintain the number of neighbors per node or the connectivity of the network, as in topology control algorithms [3].

In this paper, we analytically characterize the effective energy consumption per successfully transmitted packet in end-to-end packet transmissions in a WSN. Each node in this network could be a source, a destination, or a relay node which forwards packets from the source to the destination using the greedy forwarding routing protocol that will be reviewed later. We obtain analytical results on the average number of hops between any two nodes if the transmission is successful and the average number of hops traversed by packets before being dropped if the transmission is unsuccessful, as well as the probability of successful transmissions. A transmission is *unsuccessful* if the packet sent from a source to a destination has to be dropped at an intermediate node because the greedy forwarding routing protocol is unable to find a next-hop node that is closer to the destination. Based on the above analysis, we derive the effective energy consumption per successfully transmitted packet in end-to-end packet transmission. We show that there exists an optimum transmission range that minimizes the effective energy consumption, and give a numerical method to calculate the optimum transmission range.

Most of the previous researches in this area rely on the assumption that there exists a path between every pair of nodes which, as will be shown later, only provides an incomplete understanding on the properties of random networks. In this paper, we consider random networks in which some destinations may be unreachable from some sources using the greedy forwarding protocol. In many WSN applications, it is either impractical (due to node deployment or complicated radio environments) or unnecessary to require every node to be connected to every other node. For example, in applications which are not life-critical, e.g. habitat monitoring or environmental monitoring, it is unnecessary for all source-destination pairs to be connected. In habitat monitoring, there are scenarios where the number of objects (e.g. zebras, cane toads) being monitored is large and these objects are distributed randomly and almost independently in the surveyed region. Having a few source-destination pairs disconnected will not significantly affect the accuracy of the monitored parameter, e.g. the

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size or the density of the population [4]. In environmental monitoring, when the number of nodes used for monitoring the phenomenon or measuring the parameters is large, having a few disconnected source-destination pairs will not cause statistically significant change in the monitored parameters [5]. In addition, there is a downside on the capacity to have a high density network that is almost surely connected, because Gupta and Kumar [6] showed that as the number of nodes per unit area n increases, the throughput per source-destination pair decreases approximately like $1/\sqrt{n}$. A different point again is that [7] showed that significant energy savings can be achieved by requiring most nodes but not all nodes to be connected.

The rest of this paper is organized as follows: Section II reviews the related work. Section III introduces the network models, assumptions and definitions used in this paper. The analysis on the effective energy consumption is given in Section IV. In Section V, we show the simulation results with discussions. Our conclusions are drawn in Section VI.

II. Related Work

The topic of minimizing energy consumption in WSNs has attracted many researchers. Among them, a popular approach is to reduce energy consumption by reducing transmission power. Deng et al. [8] considered a network where nodes are Poissonly distributed in a circular area. They considered that all nodes have a common transmission range, and that there is always a path between any source and any destination using a greedy forwarding protocol. By analyzing the average progress of a packet transmitted in a single hop, they obtained an analytical result on the distance-energy efficiency, which is the ratio of the average progress to the energy consumed in a single transmission. On the basis of the above analysis, they obtained the optimum transmission range that maximizes the distance-energy efficiency for high-density networks. Recently Zhang and Gorce [9] considered the impact on energy consumption of unreliable links, where two directly connected nodes may be temporarily unreachable due to transmission errors, which is different from the disconnected paths considered in this paper.

The analysis on the energy consumption in this paper relies on the knowledge of the probability that any two nodes separated by a given Euclidean distance are k hops apart. In some literatures, two nodes are said to be k hops apart if the shortest path between them, measured by the number of hops, is k. Ta et al. [10] investigated the probability that two nodes are k hops apart for nodes Poissonly distributed in a square area. In [11] Ta et al. empirically improved the earlier result in [10] by considering the impact of boundary effect and the spatial dependence problem. The dependence problem is a major technical obstacle in the analysis of hop counts. It arises because in a WSN the probability that a randomly chosen node is k hops apart from a particular node is related to the probability that another randomly chosen node is m hops apart from the same node for $m \leq k$. This dependence problem has been largely improperly ignored in previous research.

In real applications, the path chosen for transmitting a packet between two nodes is highly dependent on the routing protocols. In view of this, some research on the probability that any two nodes are k hops apart takes the routing algorithms into consideration, and so does this paper. A popular category of routing protocols for WSNs is the greedy forwarding routing protocols. Greedy forwarding routing protocols belong to the category of geographic routing protocols [12]. In geographic routing protocols, each sensor makes routing decisions independently of other sensors by using its own location information, the location information of its neighboring sensors and the locations of the source and the destination. Geographic routing protocols have shown great potential in WSNs for having low control overhead and being adaptive to dynamic network topologies [12]. In this paper, we consider two nodes are k hops apart if the path between them, established by the greedy forwarding routing protocol, is k hops. Kuo and Liao [13] studied the probability that any two nodes are k hops apart in a network using the greedy forwarding routing protocol. They derived the hop count statistics by analyzing the average progress per hop. They tried to reduce the aforementioned dependence problem by introducing some empirical approximations. Dulman et al. [14] studied the hop count statistics by estimating the expected progress per hop using the greedy forwarding protocol. They considered the impact of the distance between nodes in the previous hop on the progress in the current hop. In this paper, we consider the impact of the distances between nodes in the previous two hops on the distance between nodes in the current hop, in a way that will give a fairly accurate result while maintaining the complexity of the analysis within control. Further we consider the possibility that a packet may be dropped due to the lack of a next-hop node. This enables our analysis to be applied for any random network, not just high density networks where there is a guaranteed path between any pair of nodes.

III. System Model

A. Network Model

In this paper, we consider a WSN located in a bounded square area. Sensors are identically and independently distributed in the area according to a homogeneous Poisson point process with a known intensity ρ.

We assume that any two nodes are directly connected if their Euclidean distance is smaller than or equal to the transmission range r_0, i.e. the unit disk communication model is considered. Every sensor has the same transmission power, and the power attenuates with distance x like x^{η}, where η is the path-loss exponent. The path-loss exponent depends on the environment and can vary from 2 in free space to 6 in urban areas [15]. Therefore the required transmission power P_t allowing a transmission range r_0 is: $P_t = Cr_0^{\eta}$, where C is a constant. Assume the time spent on transmitting a packet of unit size at a node is T_t. Then the energy consumed in transmitting a packet over a single hop is:

$$Eng(r_0) = T_t P_t + Eng_c = C_1 r_0^{\eta} + Eng_c \tag{1}$$

where $C_1 = CT_t$ is a constant and Eng_c is a constant which includes the processing and receiving power in each sensor.

B. Routing Protocol

The routing protocol has a significant impact on the number of hops between two nodes. In this paper, we consider a WSN using a greedy forwarding routing protocol. In addition to the introduction in Section II, we choose the greedy forwarding protocol that operates following two rules: 1) Every node tries to forward the packet to the node within its transmission range which is closest to the destination. 2) A packet will be dropped if a sensor cannot find a next-hop neighbor closer to the destination, and hence the transmission becomes unsuccessful.

The above rules form the basic idea of geographic random forwarding. A node may not be able to find the next hop node closer to the destination than itself because of the randomness of node deployment or a complex radio environment. A number of recovery algorithms have been proposed to route the packet around the routing void [16]. In this paper, we focus on the basic greedy forwarding protocol for analytical tractability and generality of the results. Extensions can be possibly made by considering specific route recovery algorithms.

C. Definitions of Basic Concepts and Terms

Conditioned on the Euclidean distance between the source and the destination being x_0, denote by $E_s[k_s|x_0]$ the expected number of hops for a packet to reach the destination, given the transmission is successful. Denote by $E_u[k_u|x_0]$ the expected number of hops traversed by a packet before it is dropped because no next-hop node can be found using the greedy forwarding protocol, given the transmission is unsuccessful.

We use the following example to explain the significance of the metric considered in this paper. Consider a total of N distinct source and destination pairs, where each source is separated from the associated destination by Euclidean distance x_0. Assume that each source transmits a packet of unit size to the associated destination. Therefore there are a total of N packets transmitted. Assume M packets can reach their respective destinations successfully.

The effective energy consumption per successfully transmitted packet for any pair of nodes separated by Euclidean distance x_0 is the total energy spent on transmitting all packets divided by the number of successfully received packets:

$$\begin{aligned} & Eng_{eff}(r_0|x_0) \qquad (2)\\ = & \frac{MEng(r_0)E_s[k_s|x_0] + (N-M)Eng(r_0)E_u[k_u|x_0]}{M} \\ = & Eng(r_0)\frac{Pr_s(x_0)E_s[k_s|x_0] + (1-Pr_s(x_0))E_u[k_u|x_0]}{Pr_s(x_0)} \end{aligned}$$

where $Pr_s(x_0) = \frac{M}{N}$ is the probability of successful transmissions between any pair of nodes separated by x_0.

Given the distribution of Euclidean distance between any pair of nodes $f(x_0)$, which has been given in [17], the average effective energy consumption per successfully transmitted packet in a network with transmission range r_0 is:

$$Eng_{eff}(r_0) = \int Eng_{eff}(r_0|x_0)f(x_0)dx_0 \qquad (3)$$

The effective energy consumption is a measure of the energy spent on each successfully transmitted packet. A lower effective energy consumption means a higher energy efficiency. In this paper, we use the effective energy consumption to investigate the energy efficiency in end-to-end packet transmissions.

IV. Analytical Results

In this section, by studying the distribution of the remaining Euclidean distance to the destination, we will derive the intermediate variables required in the calculation of effective energy consumption per successfully transmitted packet. We start with the calculations of the probability that any two nodes are k hops apart for $k \geq 3$ using the greedy forwarding protocol. The analysis for 1-hop and 2-hop is the same as that in previous studies [10] [14].

A. Distribution of the remaining distance

Denote by $A(x, r_1, r_2)$ the intersectional area of two circles with distance x between centers and radius r_1 and r_2 respectively. The size of the area is [18]:

$$A(x,r_1,r_2) = \qquad (4)$$
$$\begin{cases} \min(\pi r_1^2, \pi r_2^2), \quad \text{for } x \leq |r_1 - r_2| \\ r_1^2 \arccos(\frac{x^2+r_1^2-r_2^2}{2xr_1}) + r_2^2 \arccos(\frac{x^2+r_2^2-r_1^2}{2xr_2}) \\ -\frac{1}{2}\sqrt{[(r_1+r_2)^2 - x^2][x^2-(r_1-r_2)^2]}, \\ \quad \text{for } |r_1-r_2| < x < r_1+r_2 \\ 0, \quad \text{otherwise} \end{cases}$$

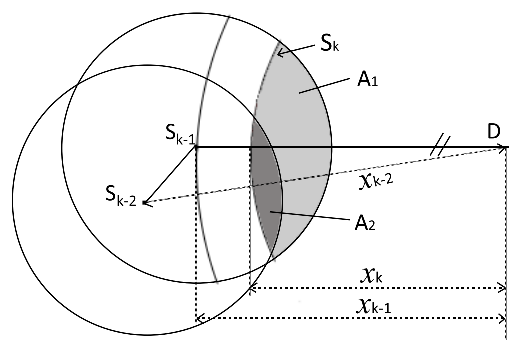

Fig. 1. Possible positions for the node at the k^{th} hop, denoted by S_k, are located on the curve, considering the positions of S_{k-1} and S_{k-2}, which are the nodes at the $k-1^{th}$ hop and $k-2^{th}$ hop from the source respectively.

Define x_k to be the remaining Euclidean distance between the k^{th} hop node (S_k) and the destination (D). As shown in Fig. 1, $A_1 = A(x_{k-1}, r_0, x_k)$ is the intersectional area of the circle centered at S_{k-1} with radius r_0 and the circle centered at D with radius x_k. Similarly we have $A_2 = A(x_{k-2}, r_0, x_k)$. Next we record the form of $\frac{\partial A(x,r_1,r_2)}{\partial r_2}$, which will be used later. From Eq. 4, for $|r_1 - r_2| < x < r_1 + r_2$, we have:

$$\begin{aligned} \frac{\partial A(x,r_1,r_2)}{\partial r_2} &= \frac{-r_1^2}{\sqrt{1-S^2}}(\frac{\partial S}{\partial r_2}) + 2r_2\arccos(T) \qquad (5)\\ &+ r_2^2(\frac{-1}{\sqrt{1-T^2}})(\frac{\partial T}{\partial r_2}) - \frac{1}{4\sqrt{W}}(\frac{\partial W}{\partial r_2}) \end{aligned}$$

$$\text{where} \quad S = \frac{x^2+r_1^2-r_2^2}{2xr_1}, \quad \frac{\partial S}{\partial r_2} = \frac{-r_2}{xr_1}$$
$$T = \frac{x^2+r_2^2-r_1^2}{2xr_2}, \quad \frac{\partial T}{\partial r_2} = \frac{r_2^2-x^2+r_1^2}{2xr_2^2}$$
$$W = ((r_1+r_2)^2-x^2)(x^2-(r_1-r_2)^2)$$
$$\frac{\partial W}{\partial r_2} = 2(r_1+r_2)(x^2-(r_1-r_2)^2) + 2((r_1+r_2)^2-x^2)(r_1-r_2)$$

Define $f(x_k,k|x_0)$ to be the joint pdf (probability density function) of the remaining Euclidean distance to the destination at the k^{th} hop node being x_k *and* the packet having been successfully forwarded k hops, conditioned on the Euclidean distance between the source and the destination being x_0. Because of the spatial dependence problem introduced in Section 1 and [11], $f(x_k,k|x_0)$ depends on the remaining distances of previous hops. The calculation of $f(x_k,k|x_0)$ will be more accurate if more previous hops are considered, but the analysis will also be more complicated. In this paper, we obtain the joint pdf by using the approximation that it only depends on previous two hops. As will be shown in Section V, this approximation gives a fairly accurate result while maintaining the complexity of the analysis under control.

Define $g(x_k,k|x_{k-1},x_{k-2},k-1)$ to be the joint pdf of the remaining Euclidean distance to the destination at the k^{th} hop node being x_k *and* the packet having been successfully forwarded k hops, conditioned on the remaining distances at the $k-1^{th}$ and $k-2^{th}$ hops being x_{k-1} and x_{k-2} respectively *and* the packet having been successfully forwarded $k-1$ hops. (Note that a packet has been successfully forwarded $k-1$ hops necessarily means that it has been successfully forwarded m hops for $m \le k-1$.) And define the corresponding cdf (cumulative distribution function) to be $Pr(X_k \le x_k, k|x_{k-1},x_{k-2},k-1)$. Ignore the boundary effect, as has been usually done in this area [11], then the cdf is equal to the probability that there is at least one node in area A_1, excluding the overlapping area $A_1 \cap A_2$ as indicated by the dark-shaded area in Fig. 1. The overlapping area $A_1 \cap A_2$ needs to be excluded because if there is a node in this area, that node will be closer to the destination than S_{k-1}, which violates the condition that S_{k-1} is the $k-1^{th}$ hop node using the greedy forwarding protocol. It is worth noting that area A_2 may not completely lie within A_1 if the position of S_{k-2} is far away from S_{k-1}. For simplicity we approximate the overlapping area $A_1 \cap A_2$ by A_2. This approximation will greatly simplify the computation while giving a sufficiently accurate result, as will be validated in Section V. Due to the Poisson distribution of sensor nodes, we have:

$$\begin{aligned} & Pr(X_k \le x_k, k|x_{k-1},x_{k-2},k-1) \\ = \;& 1-\exp(-\rho(A(x_{k-1},r_0,x_k)-A(x_{k-2},r_0,x_k))) \end{aligned} \tag{6}$$

Take the derivative of the cdf with respect to x_k, we have:

$$\begin{aligned} & g(x_k,k|x_{k-1},x_{k-2},k-1) \\ &= \frac{\partial Pr(X_k \le x_k,k|x_{k-1},x_{k-2},k-1)}{\partial x_k} \\ &= \rho\left(\frac{\partial A(x_{k-1},r_0,x_k)}{\partial x_k} - \frac{\partial A(x_{k-2},r_0,x_k)}{\partial x_k}\right) \\ &\quad \times \exp\left(-\rho(A(x_{k-1},r_0,x_k)-A(x_{k-2},r_0,x_k))\right) \end{aligned} \tag{7}$$

where $\frac{\partial A(x_{k-1},r_0,x_k)}{\partial x_k}$ and $\frac{\partial A(x_{k-2},r_0,x_k)}{\partial x_k}$ can be calculated using Eq. 5.

Define $h(x_k,x_{k-1},k|x_0)$ to be the joint pdf of the remaining Euclidean distances at the k^{th} hop node and $k-1^{th}$ hop node being x_k and x_{k-1} respectively *and* the packet having been successfully forwarded k hops, conditioned on the distance between the source and destination being x_0.

For $k=1$, the remaining distance at the previous hop node is the distance between the source and destination. Therefore:

$$f(x_1,1|x_0) = \rho\frac{\partial A(x_0,r_0,x_1)}{\partial x_1}e^{-\rho A(x_0,r_0,x_1)} \tag{8}$$

Based on the above result, for $k=2$ we have:

$$h(x_2,x_1,2|x_0) = g(x_2,2|x_1,x_0,1)f(x_1,1|x_0) \tag{9}$$

For $k>2$, $h(x_k,x_{k-1},k|x_0)$ can be calculated recursively:

$$h(x_k,x_{k-1},k|x_0) = \int_{r_0}^{x_0} g(x_k,k|x_{k-1},x_{k-2},k-1)\, h(x_{k-1},x_{k-2},k-1|x_0)dx_{k-2} \tag{10}$$

Finally for $k>1$ we have:

$$f(x_k,k|x_0) = \int_{r_0}^{x_0} h(x_k,x_{k-1},k|x_0)dx_{k-1} \tag{11}$$

B. Average number of hops for successful transmission

Define $\phi(k|x_0)$ to be the probability that the destination can be reached at the k^{th} hop by the greedy forwarding protocol given that the Euclidean distance between source and destination is x_0. By the assumption of unit disk model, the destination can be reached at the k^{th} hop if the remaining distance at the $k-1^{th}$ hop node is within the transmission range of the destination. Therefore:

$$\phi(k|x_0) = \int_0^{r_0} f(x_{k-1},k-1|x_0)dx_{k-1} \tag{12}$$

Denote by $\phi_s(k|x_0)$ the conditional probability that a packet reaches its destination at the k^{th} hop, conditioned on the transmission being successful *and* the Euclidean distance between source and destination being x_0. It is trivial to see that $\phi(k|x_0) = \phi_s(k|x_0)Pr_s(x_0)$, where $Pr_s(x_0)$ is the probability of successful transmissions for any pair of nodes separated by Euclidean distance x_0. Therefore:

$$\begin{aligned} \sum_{k_s=1}^{\infty} k_s\phi(k_s|x_0) &= Pr_s(x_0)\sum_{k_s=1}^{\infty} k_s\phi_s(k_s|x_0) \\ &= Pr_s(x_0)E_s[k_s|x_0] \end{aligned} \tag{13}$$

where $E_s[k_s|x_0]$ is defined in Section III.D. In reality, an upper bound on k_s can be found such that $\phi(k_s|x_0)$ is 0 for k_s greater than the upper bound. So Eq. 13 and other equations in a similar form only need to be computed for a finite range of k_s.

C. The probability of successful transmissions

An end-to-end transmission is successful if the destination is reachable by any number of hops. Therefore:

$$Pr_s(x_0) = \sum_{k_s=1}^{\infty} \phi(k_s|x_0) \quad (14)$$

D. Average number of hops for unsuccessful transmission

Define $Pr(k|x_0)$ to be the probability of a packet having been successfully forwarded k hops from the source toward the destination x_0 apart, but *not* reaching the destination in k hops, which distinguishes $Pr(k|x_0)$ from $\phi(k|x_0)$. Therefore:

$$Pr(k|x_0) = \int_0^{x_0} f(x_k, k|x_0)dx_k \quad (15)$$

Based on the example introduced in Section III.D, we further assume that only M_k packets reach the k^{th} hop nodes. Then $Pr(k|x_0) = \frac{M_k}{N}$. At the next hop, there are three possibilities for these M_k packets: 1) W_{k+1} packets can reach the destination at $k+1^{th}$ hop; 2) M_{k+1} packets cannot reach the destination at $k+1^{th}$ hop but can be forwarded to $k+1^{th}$ hop neighbors; 3) Other packets cannot be forwarded to $k+1^{th}$ hop neighbors and are dropped.

Define $\psi(k_u|x_0)$ to be the probability of the packets being dropped at the k_u^{th} hop due to the lack of a next-hop neighbor using the greedy forwarding protocol. Then:

$$\psi(k_u|x_0) = \frac{M_{k_u} - M_{k_u+1} - W_{k_u+1}}{N}$$
$$= Pr(k_u|x_0) - Pr(k_u+1|x_0) - \phi(k_u+1|x_0) \quad (16)$$

The average number of hops for unsuccessful transmissions between a source and a destination separated x_0 apart is the expected value of k_u whose pdf is given by $\psi(k_u|x_0)$. Similar to Eq. 13, we have:

$$\sum_{k_u=1}^{\infty} k_u \psi(k_u|x_0) = (1 - Pr_s(x_0))E_u[k_u|x_0] \quad (17)$$

E. Effective energy consumption

Given the above analysis, the effective energy consumption per successfully transmitted packet can be computed using Eq. 3 where the relevant values are given in Eq. 13, 14 and 17 respectively. The minimum value of effective energy consumption and the optimum transmission range can be found by taking the derivative of Eq. 3 with respect to r_0 and setting the derivative to 0 to solve the equation. Due to the complexity of the analytical expressions, we will only present numerical results in the rest of this paper.

V. Simulation Results

The simulations are conducted in a simulator written in C++ so that we have full controls on the program complexity and efficiency. We deploy sensors in a 400×400 square area according to a homogeneous Poisson process with intensity $\rho = 0.002$. The route between two nodes is determined by the greedy forwarding routing algorithm introduced in Section III-B, without the approximations used in the analytical analysis. Therefore the simulation results provide a useful gauge on the accuracy of our analytical analysis. The transmission range is varied from 12 to 60, which causes the average node degree $\rho\pi r_0^2$ to vary from around 1 to 23. The constants in Eq. 1 are $\eta = 2$, $C_1 = 0.01$ and $Eng_c = 0.01$, where Eng_c is found to have very limited impact on the results, which agrees with [19]. Every point shown in the plot is the average value from 2000 simulations. As the number of instances of random networks used in the simulation is large, the confidence interval is too small to be distinguishable and hence is ignored in the following plots.

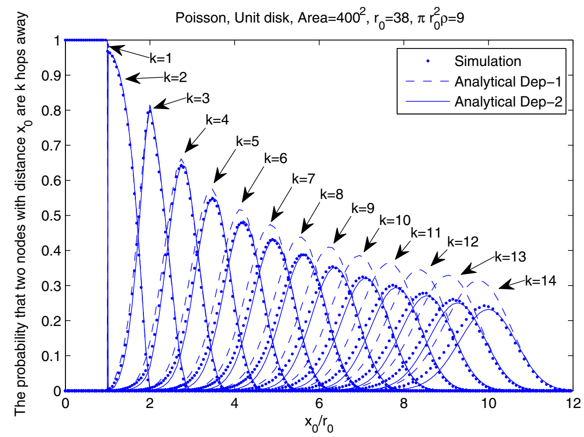

Fig. 2. The probability that any two nodes separated by Euclidean distance x_0 are k hops apart. Dep-1 and Dep-2 stand for the analytical results calculated by considering the dependency on previous one hop [8] and previous two hops respectively, where the latter analysis is introduced in this paper.

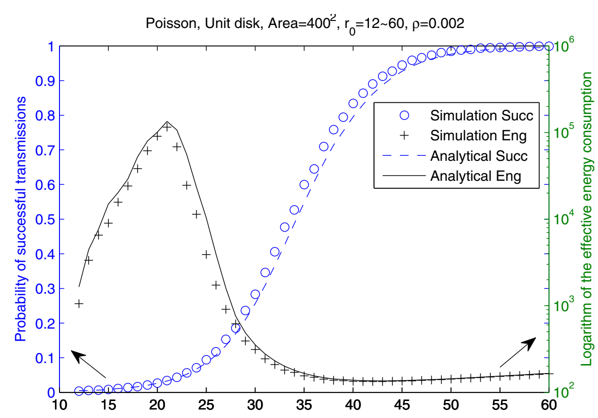

Fig. 3. Simulation and analytical results on the probability of successful transmissions (Succ) and the effective energy consumption (Eng).

Fig. 2 illustrates the improvement of accuracy on the probability that any two nodes are k hops apart. In this paper, we considered that the remaining Euclidean distance at the k^{th} hop node depends on the remaining Euclidean distance at two previous hop nodes. Obviously if dependence on a larger number of previous hops is considered, the result will be more accurate (and the analysis will also be more complicated). Fig. 2 shows however the improvement in accuracy by considering previous m hops for $m > 2$ will be marginal.

Fig. 3 shows the probability of successful transmissions and average effective energy consumption per successfully transmitted packet in a network. It can be seen that, unsurprisingly, the probability of successful transmissions increases from nearly 0 to nearly 1 as the transmission range r_0 increases. In contrast, the average effective energy consumption could hardly have been predicted by heuristic reasoning, and needs more explanations. When r_0 is small, the network is made of a large number of small components (a set of nodes where any two of them are connected to each other by paths). An increase in r_0 will cause an increase in the size of these components. Therefore the average number of hops for unsuccessful transmission increases and the energy wasted on unsuccessful transmission also increases. As r_0 further increases, although the average number of hops for successful/unsuccessful transmission still increases, the energy wasted on unsuccessful transmission starts to decrease as more source-destination pairs become connected. The balance of the two effects causes the effective energy consumption to peak at $r_0 \approx 22$. Above this r_0, the decrease in wasted energy starts to dominate, which causes a subsequent decrease in effective energy consumption. As r_0 increases further, the average number of hops approaches its maximum and the energy wasted on unsuccessful transmission also reduces to a small amount. These cause the effective energy consumption to reach its minimum at $r_0 \approx 43$. Above this transmission range, most source-destination pairs are connected as shown in Fig. 3. Another effect starts to dominate. That is, the increase in r_0 causes the increase in the per-hop energy consumption (like r_0^2) and the decrease in the number of hops (approximately like $1/r_0$). The net effect is an increase in the effective energy consumption with the increased r_0. Most previous studies only considered this last stage and therefore cannot give a complete understanding of the hop count statistics and energy consumption in end-to-end packet transmissions.

It is interesting to note that the energy optimizing transmission range is 43, which corresponds to a network with most (around 90%) source-destination pairs connected but not all of them. (Note that for $r_0 < 10$, most source-destination pairs are disconnected and no meaningful service can be provided by the WSN.) In order for more than 99% source-destination pairs to be connected, r_0 has to be larger than 55 and the effective energy consumption will increase by at least 13% compared with its minimum value. This observation also agrees with the analytical results in [7], which investigated the energy required for a large percentage of nodes in a network to be connected.

VI. CONCLUSION

In this paper, we investigated energy consumed in end-to-end packet transmissions in a WSN using a greedy forwarding routing protocol. We obtained analytical results on the average number of hops between any two nodes if the transmission is successful and the average number of hops traversed by packets before being dropped if the transmission is unsuccessful. We further derived the effective energy consumption per successfully transmitted packet. We showed the existence of an optimum transmission range which minimizes the effective energy consumption, and the method to compute such optimum transmission range. The research in this paper provides guidelines on choosing the transmission range for an energy-efficient WSN and sheds insight on how energy consumption varies with the transmission range.

In our future work, we will consider a more realistic radio environment, e.g. the log-normal shadowing model and the time variation of the links. In addition, the impact on the optimum transmission range of the path loss exponent and sensor density can be further investigated.

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